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Effect of threshold stress processes on ductility

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When a threshold stress process influences the creep behaviour, the mechanical deformation of the material is not driven by the total applied stress but rather by an effective stress. The effect of the operation of such a process on ductility is examined using the concept of plastic instability. Also, this effect is compared with that caused by the operation of two sequential processes (slower process controls). Based on this comparison, it is suggested that a distinction between the operation of a threshold stress process and the operation of two sequential processes on the basis of ductility tests may be difficult, especially when cavitation occurs at low stresses or when an independent interaction exists between a threshold stress process and a low-stress deformation mechanism.

1. Introduction

Under conditions of steady-state deformation, the creep rate, $\dot{\epsilon}$, may generally be represented by an expression of the form

$$\dot{\epsilon} = B\frac{\sigma^n}{d^s}\exp(-Q/RT), \tag{1}$$

where B is a constant, σ is the applied stress, n is the stress exponent, d is the grain size, s is the grain size sensitivity, Q is the apparent activation energy, R is the gas constant and T is the absolute temperature. Equation 1 is referred to as the creep power law and its validity, when expressed in a normalized form [1], has been established for a wide range of materials.

Rate controlling mechanisms of creep are generally identified by comparing the experimentally measured values of n, Q, and s with those values established for various basic processes. In addition, when interaction between two different creep processes is significant over certain ranges of experimental conditions, the nature of the interaction can be inferred from examining the variation in the stress exponent, n, as a function of the applied stress, σ, and/or the variation in the activation energy, Q, as a function of the absolute temperature [2]. For instance, a sequential interaction between two processes (the slower process controls) results in an increase in the stress exponent, n, with decreasing stress whereas the opposite is true for the case of an independent interaction between two processes (the faster process controls). While many examples are available in the creep literature to illustrate the applicability of this simple analysis, recent considerations [3, 4] suggest that under certain experimental conditions the stress exponent, n, and the activation energy for creep, Q, may not provide sufficiently good criteria to distinguish the difference between sequential processes and threshold stress processes; a threshold stress process signifies that an effective stress, σ_e ($\sigma_e = \sigma - \sigma_0$, where σ_0 is the stress needed for the onset of deformation), rather than the applied stress (Equation 1) is responsible for the measured strain rate. This difficulty in distinguishing the difference between the operation of a threshold stress process and the operation of two sequential processes on the basis of the stress exponent, n, arises basically from the close similarities of $\log\dot{\epsilon} - \log\sigma$ plots (from which n is inferred) for the two types of deformation processes. Also, when a threshold stress process depends strongly on temperature, the operation of such a process results in Arrhenius plots of $\log\dot{\epsilon}$ against $1/T$ (from which Q is inferred) similar to those

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produced by the operation of two sequential processes.

In view of the frequent application of the concepts of sequential processes and threshold stress processes to the interpretation of low-stress creep data and to the development of constitutive equations, it seems desirable to examine whether the close resemblance between the two types of processes is only confined to the stress exponent, n, and the activation energy for creep, Q, or may extend to include other mechanical parameters. Of these mechanical parameters, plastic instability appears to be an appropriate choice partly due to its relevance to the process of high-temperature flow and fracture and partly because of its recent utilization to examine the low-stress creep behaviour of some materials [5, 6]. This paper therefore examines the effect of the operation of a threshold stress process on plastic instability and compares this effect with that arising from the operation of two sequential processes.

2. Analysis and discussion

As a first step in the analysis, it is essential to introduce the following two basic equations:

$$\frac{dA}{dt} = -A\dot{\epsilon} \tag{2}$$

and

$$P = \sigma A, \tag{3}$$

where A is the instantaneous area, dA/dt is the rate change of the area, and P is the applied load. Equation 2 is the continuity equation, which incorporates the assumption of constancy of volume, and Equation 3 defines the applied stress.

For simple creep power-law behaviour that can be represented by Equation 1, it is well documented that the rate of change of the area, $dA/dt(\dot{A})$, is related to the instantaneous area, A, and the stress exponent, n, by the following expression [7]:

$$\left|\frac{dA}{dt}\right| = A\dot{\epsilon} = C(P)^n/A^{n-1}, \tag{4}$$

where C is a constant for constant temperature and grain size; according to Equation 1, $C = (B/d^s)\exp(-Q/RT)$. While the approach taken to arrive at Equation 4 is rather simple, the expression may serve as a starting point in the present analysis. A simple form of plot that represents Equation 4 is shown in Fig. 1, where $\log|dA/dt|$ is plotted against $\log A$; because of the selection of a double logarithmic scale, this plot is different from that reported elsewhere [7] for Equation 4 and produces a straight line having a slope of $1-n$. The straight line shown in Fig. 2 has two parameters,

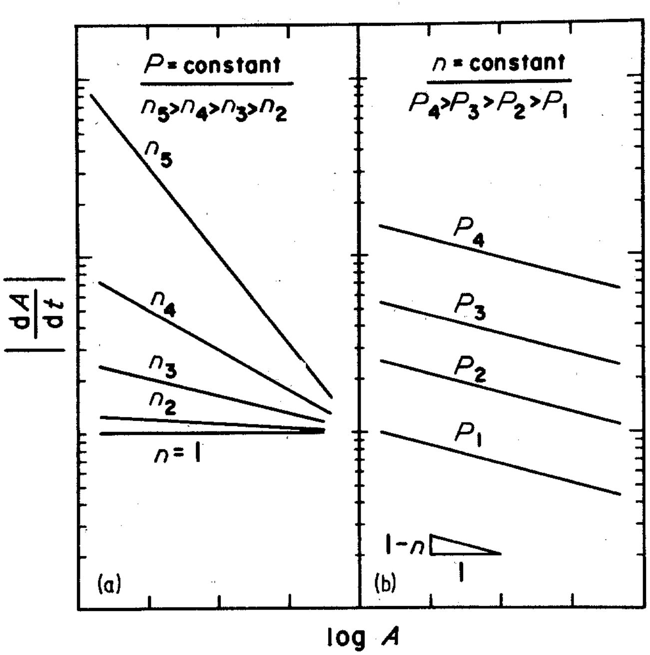

Figure 1 The log of the rate of change of area, dA/dt, against the log of the instantaneous area, A, for (a) a constant value of the applied load, P, and (b) a constant value of the stress exponent, n.

P and n, and accordingly offers two relationships: (a) For a constant value of P, a family of non-parallel lines is produced with changing the stress exponent (Fig. 2a), and (b) for a constant value of n, a family of parallel lines is obtained as P varies (Fig. 2b). While the first plot (Fig. 2a) indicates that the rate growth of necks is drastically accelerated as the stress exponent, n, increases, the second plot signifies that the load level has no effect on the shape states of a neck which develop in a tensile specimen prior to fracture and that only the time required to reach a specific state is shortened by increasing P, i.e., in the absence of cavitation, no variation in ductility occurs with changing the load level. The situations for sequential processes and threshold stress process are now examined in the light of Equation 1 and Fig. 1.

2.1. Sequential processes

Deformation processes may operate either independently (so that the fastest controls) or sequentially (so that the slowest controls). For two processes, Processes a and b, operating sequentially in such a way that each process participates for a different time through any period, t, and contributes an identical strain to preserve the integrity of the material, the total creep rate, $\dot{\epsilon}$, is given by [2]

$$\dot{\epsilon} = \frac{\dot{\epsilon}_a \dot{\epsilon}_b}{\dot{\epsilon}_a + \dot{\epsilon}_b}. \tag{5}$$

To apply Equation 4 to the sequential interaction and also to provide a comparison, later on, with the behaviour of a threshold stress process, we shall assume that the two sequential processes are represented by the following constitutive equations:

$$\dot{\epsilon}_a = B_a \sigma^{n_a} \exp(-Q_a/RT) \tag{6a}$$

and

$$\dot{\epsilon}_b = B_b \sigma^{n_b} \exp(-Q_b/RT). \tag{6b}$$

The values of B_a, n_a, Q_a, B_b, n_b, and Q_b have been selected as: 9.4×10^{-3}, 2, $84\,\text{kJ mol}^{-1}$, 10^5, 5 and $209\,\text{kJ mol}^{-1}$, respectively. Consideration of Equation 5 along with Equation 6 shows that Process a, having a stress exponent of 2, controls the creep behaviour of the material at high stresses and that Process b, having a stress exponent of 5, is the dominant process at low stresses.

By using Equation 6 to express $\dot{\epsilon}_a$ and $\dot{\epsilon}_b$ in Equation 5, replacing $\dot{\epsilon}$ in Equation 2 by $\dot{\epsilon}$ of

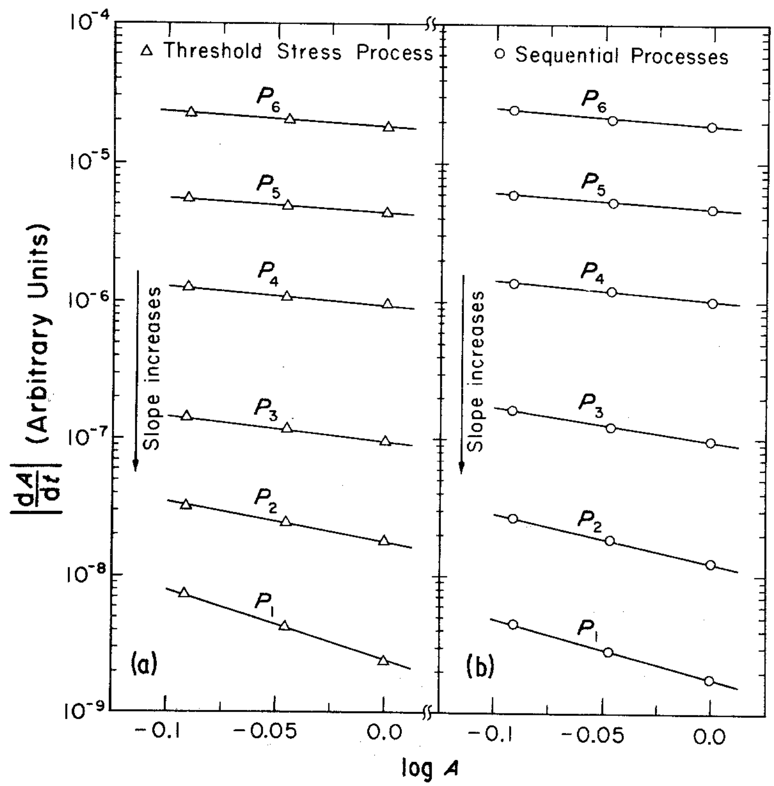

Figure 2 The log of the rate of change of area, dA/dt, against the log of the instantaneous area, A, for (a) a power-law threshold stress process and (b) two sequential processes.

Equation 5, and writing $\sigma = P/A$ (Equation 3), the relationship between dA/dt and A can be given by the form:

$$\left|\frac{dA}{dt}\right| = \frac{k_a k_b P^{n_b}}{k_a A^{n_b-1} + k_b A^{n_a-1} P^L}, \quad (7)$$

where

$$k_a = B_a \exp(-Q_a/RT),$$

$$k_b = B_b \exp(-Q_b/RT)$$

and

$$L = n_b - n_a.$$

The variation in $\log|dA/dt|$ as a function of $\log A$ for sequential Processes a and b is plotted in Fig. 2 for six different values of P and for a given range of A*. It is clear that the plot of Fig. 2 results in a series of straight lines for the range selected for A, with each straight line having a slope of $1-n$, and that these lines combine the features of both plots of Fig. 1. For high values of P, i.e., $P > P_6$, Process a is dominant and the lines are essentially parallel with a slope of -1, and for low values of $P, P < P_1$, Process b is dominant and the lines exhibit a slope of -4. By contrast, lines obtained for intermediate loads are non-parallel and the slope changes continuously from a limiting value of -1 to a limiting value of -4. This change in slope from -1 to -4 with decreasing P indicates, according to Fig. 1, that inhomogeneous deformation becomes increasingly localized as the creep behaviour changes from that typical of Process a to that typical of Process b.

2.2. A threshold stress process

Let us consider a hypothetical situation in which modified power-law creep, associated with a threshold stress, controls the creep behaviour and obeys the following empirical equation:

$$\dot{\epsilon} = C_{ts}(\sigma - \sigma_0)^n \exp(-Q/RT),$$

with (8)

$$\sigma_0 = C_0 \exp(Q_0/RT),$$

where C_{ts} and C_0 are constants, and Q_0 is an activation energy associated with σ_0. Although there is no theoretical justification for a strong temperature dependence of σ_0, as assumed by Equation 8, recent experimental evidence indicates that this situation is not unrealistic. For example, experimental measurements made during superplastic flow of a duplex stainless steel [8] have shown the presence of a threshold stress that decreased with increasing temperature. In addition, low-stress creep data of several systems, as examined by Burton [9], suggest the presence of a threshold stress that depends strongly on temperature during diffusion creep ($\dot{\epsilon} \propto \sigma - \sigma_0$). The values of C_{ts}, n, Q, C_0, and Q_0 have been selected as 9.4×10^{-3}, 2, 84, 0.01 and $33\,\mathrm{kJ\,mol^{-1}}$, respectively. This choice of Q, Q_0, C_{ts} and C_0 shows that the creep rates due to the threshold stress process, when plotted against the applied stress on a logarithmic scale in Fig. 3, are experimentally indistinguishable from those ascribable to the sequential summation of Equations 5 and 6 over the same experimental stress range and for four different temperatures.

When Equation 8, under constant temperature condition, is combined with Equations 2 and 3, the variation in $|dA/dt|$ as a function of A for a threshold stress process can be expressed as

$$\left|\frac{dA}{dt}\right| = C_2 P^n \left(1 - \frac{\sigma_0 A}{P}\right) \Big/ A^{n-1}, \quad (9)$$

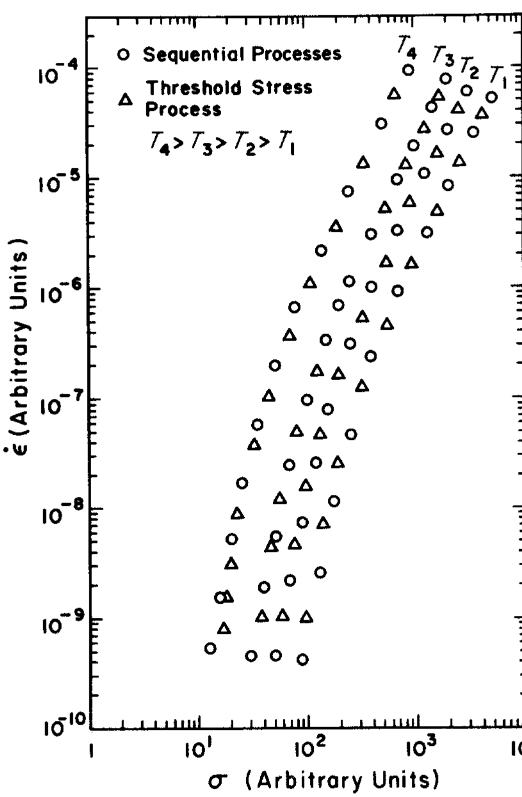

Figure 3 Log of the strain rate against log of the applied stress at four temperatures for a power-law threshold stress process and two sequential processes.

*The range of A is selected so that the different areas of the tensile specimen exhibit almost the same value of n during deformation.

where $C_2 = C_{ts} \exp(-Q/RT)$. Using the selected values of n, Q, Q_0, C_{ts} and C_0, $\log |dA/dt|$ is plotted against $\log A$ in Fig. 2a for six different load levels which are comparable with those used in plotting the sequential situation (Fig. 2b). An examination of this plot suggests two points. Firstly, for high values of P ($P > P_6$), the slope of each corresponding line, like that due to the sequential summation, is -1 and σ_0 has no effect on $|dA/dt|$. Secondly, decreasing the value of P causes the slope of the line to increase, a trend which is also similar to that exhibited by the sequential summation. The similarities between the plot of $\log |dA/dt|$ against $\log A$ for a threshold stress process and that for the sequential summation suggest that a threshold stress process, like the sequential summation, would lead to a decrease in ductility with decreasing the applied load. This suggestion is examined in the next section.

2.3. Ductility

A number of investigators [10–16] have developed equations that correlate the elongation at fracture, e_f, with the stress exponent, n, using two different types of treatment. In the first type of treatment [10, 11], the ductility equation was derived by using a force balance between the homogeneous and imperfect regions of the specimen and assuming that the strain in the imperfect region approaches infinity as the strain in the homogeneous region reaches the fracture strain; whereas in the second type of treatment [11–16], the original Hart analysis [17] for the rate of development of an inhomogeneity in the cross-sectional area of a tensile specimen was extended* to predict ductility equations. Regardless of the type of treatment chosen, the development of ductility expressions has required the use of constitutive equations that can predict or describe the mechanical behaviour of the material. Three commonly used equations for this purpose are: $d \ln \sigma = m \, d \ln \dot{\epsilon} + \gamma \, d\epsilon$, $\sigma = k\epsilon^N \dot{\epsilon}^m$ and the reduced steady-state form $\sigma = k\dot{\epsilon}^m$, where m is the strain-rate sensitivity ($m = 1/n$ under steady-state conditions), γ is the work hardening coefficient and $N = \gamma/\epsilon$. Use of these equations, however, leads to analytical solutions only when simplifying assumptions are incorporated into the analysis. The constitutive equations under consideration in the present work (Equations 5, 6 and 8) are clearly different from those commonly used and, because of this difference, difficulties may be encountered in the analysis predicting ductility if a general solution is attempted. Accordingly, in conducting the present investigation, it seems desirable, for the purpose of avoiding analytical complications, to adopt a linealized approach, along the lines suggested by Hart [17], which can predict to a first approximation the effects of the newly considered constitutive equations on the fracture strain of a tensile specimen.

We shall consider a tensile test specimen that is deforming under constant load, P, during steady-state conditions (strain-hardening is absent). The specimen has initially a uniform cross-sectional area A (homogeneous region) except for a short length in which the cross-sectional area differs by a small quantity δA (imperfect region). Also, it is assumed that the specimen is deforming in uniaxial tension (triaxially due to necking is ignored). After application of load, P, for a time interval, the cross-sectional areas in the homogeneous and imperfect regions are A and $(A + \delta A)$, respectively. With these descriptions and assumptions, which are similar to those included in the analysis of Hart [17], the dependence of δA on δA_0, A, and A_0 can be analysed for the two sequential processes given by Equations 5 and 6 and the threshold stress process represented by Equation 8.

As a first step, we apply Equations 2 and 3 to both the homogeneous and imperfect regions of the specimen. After neglecting products of small quantities, we obtain the following two relations:

$$\frac{d\,\delta A}{dt} = -(A\delta\dot{\epsilon} + \dot{\epsilon}\delta A) \tag{10}$$

and

$$\frac{\delta\sigma}{\sigma} = -\frac{\delta A}{A}\,. \tag{11}$$

Further progress beyond these two equations requires consideration of the appropriate constitutive equations.

2.3.1. Sequential processes

The total creep rate, $\dot{\epsilon}$, arising from two processes, a and b, acting sequentially, and satisfying conditions state in Section 2.1, is given by Equation 5 which can, using the relationships $\dot{\epsilon}_a = k_a\sigma^{n_a}$

*The assumption that the strain in the inhomogeneity is infinite at fracture is also incorporated into the analysis of one investigation [13].

$[k = B_a \exp(-Q_a/RT)]$ and $\dot{\varepsilon}_b = k_b\sigma_b^n [k_b = B_b \exp(-Q_b/RT)]$, be written in the form:

$$\dot{\varepsilon} = \frac{k_a k_b \sigma^{n_a+n_b}}{k_a\sigma^{n_a} + k_b\sigma^{n_b}} \quad \text{for } T = \text{constant}. \quad (12)$$

If this equation is applied to both the homogeneous and imperfect regions, one obtains, after some simplifications, the approximate form:

$$\frac{\delta\dot{\varepsilon}}{\dot{\varepsilon}} \simeq \left(\frac{n_a k_b \sigma^{n_b} + n_b k_a \sigma^{n_a}}{k_a\sigma^{n_a} + k_b\sigma^{n_b}}\right)\frac{\delta\sigma}{\sigma}. \quad (13)$$

Combining Equations 10, 11 and 13, rearranging, and using Equation 3 ($P = \sigma A$), we find

$$\frac{\mathrm{d}\,\delta A}{\delta A} = \left(1 - \frac{n_a k_b P^L + n_b k_a A^L}{k_a A^L + k_b P^L}\right)\frac{\mathrm{d}A}{A}, \quad (14)$$

where $L = n_b - n_a$. Under the condition of constant load, Equation 14 can be integrated from A_0 to A to give

$$\frac{\delta A}{\delta A_0} = \left(\frac{A}{A_0}\right)^{1-n_a}\left(\frac{k_a A_0^L + k_b P^L}{k_a A^L + k_b P^L}\right). \quad (15)$$

An examination of Equation 15 reveals two limits which are consistent with both the original Hart analysis [17] and the characteristics of the sequential summation [2]. First, for very high values of P, the terms $k_a A_0^L$ and $k_a A^L$ are very small compared with $k_b P^L$ and as a result Equation 15 reduces to

$$\frac{\delta A}{\delta A_0} \simeq \left(\frac{A}{A_0}\right)^{1-n_a}. \quad (16)$$

Equation 16 is the form obtained from Hart's analysis [17] under the condition of the operation of Process a. Second, for very small values of P, $k_a A_0^L$ and $k_a A^L$ become dominant terms and Equation 15 can be written as

$$\frac{\delta A}{\delta A_0} \simeq \left(\frac{A}{A_0}\right)^{1-n_a}\left(\frac{A_0}{A}\right)^L$$

$$= \left(\frac{A}{A_0}\right)^{1-n_b}. \quad (17)$$

Again this form represents the result of Hart's analysis [17] when Process b controls the plastic flow of the material.

Multiplying both sides of Equation 15 by A_0/A and replacing A_0/A by $1 + e$, where e is the average engineering strain, the equation

$$\frac{(\delta A/A)}{(\delta A_0/A_0)} = (1+e)^{n_a}\left[\frac{k_a A_0^L + k_b P^L}{k_a(A_0/1+e)^L + k_b P^L}\right] \quad (18)$$

is obtained. It is assumed that fracture occurs when $(\delta A/A)/(\delta A_0/A_0)$ approaches a critical value, K_c [13]. With this condition, Equation 18 can be expressed as

$$K_c = (1+e_f)^{n_a}\left[\frac{k_a A_0^L + k_b P^L}{k_a(A_0/1+e_f)^L + k_b P^L}\right]. \quad (19)$$

It is recognized that K_c could be a function of many variables including stress exponent, shape and size of samples, and other substructural features, but a recent investigation [13] shows that as a first approximation K_c is a function of the stress exponent, n, and exhibits two bounds: $K_c = 200$ for $n = 2$ and $K_c = 4$ for $n = 100$. However, since the present analysis is intended to provide a comparison between two types of behaviour (sequential and threshold stress processes), it seems reasonable to use an average value of 100 for K_c. Using this value of K_c and taking $A_0 = 1$, Equation 19 is solved graphically for different values of P (for the same range used in Fig. 2). The results of the solution of Equation 19 are shown in Fig. 4b, where the elongation to fracture, e_f, is plotted against P on a logarithmic scale. As expected, e_f decreases with P from an upper limit determined by Process a ($n = 2$) to a lower limit determined by Process b ($n = 5$).

2.3.2. A threshold stress process

Using the constitutive law of the threshold stress process given in Equation 8, the following steps similar to those carried out for the sequential summation, the dependence of δA on δA_0, A and A_0 can be expressed as

$$\frac{\mathrm{d}\delta A}{\delta A} = \left(1 - \frac{n}{1 - A\sigma_0/P}\right)\frac{\mathrm{d}A}{A}. \quad (20)$$

Upon integration from A_0 to A (P is constant and $P/A > \sigma_0$), it is possible to write the result in the form of Equation 15 as

$$\frac{\delta A}{\delta A_0} = \left(\frac{A}{A_0}\right)^{1-n}\left(\frac{A\sigma_0 - P}{A_0\sigma_0 - P}\right)^n. \quad (21)$$

When $P \gg A_0\sigma_0$, Equation 21 reduces to the form obtained from Hart's analysis [17]. Multiplying both sides of Equation 21 by A_0/A, and setting

$(\delta A/A)/(\delta A_0/A) \rightarrow K_c$ as $e \rightarrow e_f$, the expression of the elongation to fracture under the condition of the operation of a threshold stress process can be given by

$$K_c = \left\{ (1 + e_f) \left[\frac{(\sigma_0 A_0 / 1 + e_f) - P}{\sigma_0 A_0 - P} \right] \right\}^n . \quad (22)$$

Equation 22 is solved graphically to obtain e_f as a function of P using the same conditions chosen for the sequential summation (range of P, temperature, A_0, K_c, etc.); σ_0 is calculated from Equation 8.

The values of e_f estimated from Equation 22 are plotted against P on a logarithmic scale in Fig. 4a. A comparison between the trend shown by the threshold stress process (Fig. 4a) and that exhibited by the sequential summation (Fig. 4b) reveals two similarities and one difference. Both trends are similar with respect to (a) the asymptotic value of e_f at the highest value of P, and (b) the values of e_f at intermediate values of P (the lowest value is indicated by an arrow) which, in fact, represent the transition domain of the two sequential processes. These similarities, in turn, indicate that both types of processes would lead to a decrease in ductility with decreasing loads. In contrast, the difference between both trends (e_f against P) is manifested by the observation that the threshold stress process represented by Equation 8, unlike the sequential summation, does not result in an asymptotic value of e_f at very low values of P; this is expected since the value of the stress exponent inferred from Equation 8 increases continuously with σ. While this difference in trend can be used to distinguish between the two types of deformation processes, several complications, arising from factors which are not considered in the solution of Equations 14 and 20, may interfere and mask the trend. Among these factors are the possibility of nucleation and growth of cavities under the conditions of low stresses and long testing time and /or the possibility of intervention by an independent mechanism at very low stresses. The nucleation and growth of cavities, when combined with the operation of two sequential processes, could lead to a continuous decrease in ductility with decreasing the applied load, a variation which simulates that arising from a threshold stress process as shown by Fig. 4a. On the other hand, the possibility of an interaction between a threshold stress process and a second, independent low-stress mechanism could result in an enhancement of ductility at low stresses. This enhancement, in turn, could produce a trend which is similar to that produced by the operation of two sequential processes.

Finally, there is one comment concerning the present analysis and the assumptions involved. In developing the ductility expressions (Equations 19 and 22), it is implicitly assumed that the value of the stress exponent is the same in both homogeneous and imperfect regions of the tensile specimen. This assumption seems unrealistic for very large strain increments, since the operation of either sequential processes or threshold stress processes would result in a variation in the stress exponent, n, with position along a tensile specimen. However, the major part of the elongation to fracture is obtained when the neck is still very small, implying that the difference between the value of n in the homogeneous region and that in the imperfect region during most of the ductility test may be so small that the assumption of constancy of n may not be seriously violated. In addition, the fact that several ductility expressions [10–16] tend to predict accurately the elongation to fracture [18] even in the presence of strain-

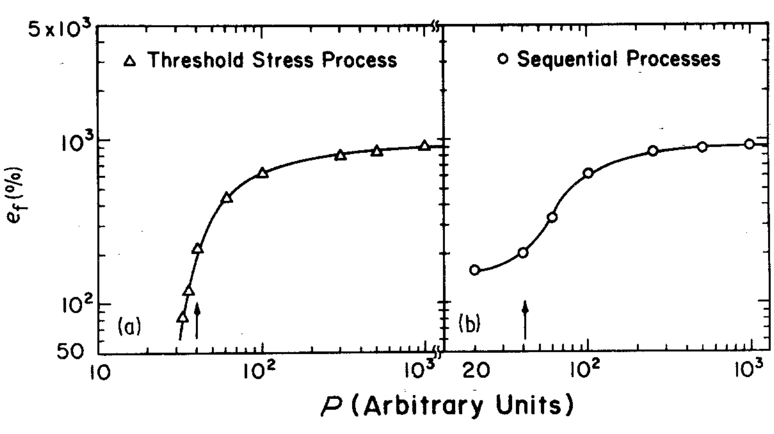

Figure 4 The elongation to fracture, e_f, against the applied load, P, for (a) a power-law threshold stress process, and (b) two sequential processes.

hardening (n is a function of strain), and even when a variation in the size of the imperfection is considered, suggests that the assumption of constancy of n along the specimen as well as the assumptions of a uniaxial stress state and homogeneous deformation may not strongly affect the prediction of ductility in the large strain limit.

3. Conclusions

It is shown that the correlation between the stress exponent and ductility may not provide a conclusive distinction between the operation of a threshold stress process and the operation of two sequential processes because of the similarity of ductility plots for both cases. Also, the occurrence of extensive cavitation at low stress and/or the presence of other independent low-stress mechanisms could contribute to the difficulty of such a distinction.

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